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Fractional-order modeling and optimal control of corruption dynamics in Police and Judiciary Systems

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Abstract
sThis study explores corruption dynamics in police and judiciary systems using Fractional Order Differential Equations (FODEs), which capture memory and hereditary effects, offering deeper insight into corruption progression. Integrating Optimal Control Theory, the research designs feasible intervention strategies under real-world constraints. An FODE-based model is developed, and numerical simulations, combined with sensitivity analysis, identify parameters most influencing corruption, guiding targeted interventions. Optimized strategies, including public sensitization, education campaigns, and punitive measures against corrupt officials, are assessed for effectiveness. Results show that combining these interventions substantially reduces corruption compared to uncontrolled scenarios. The study emphasizes multi-pronged, tailored strategies and demonstrates the value of fractional-order modeling for understanding complex socio-institutional phenomena. Findings provide actionable guidance for policymakers to implement effective anti-corruption measures and establish a foundation for future research aimed at refining and extending these strategies for more adaptive and evidence-based governance frameworks.

Keywords: Corruption dynamics, Fractional Order Differential Equations (FODEs), Optimal Control, Police system, Judiciary system, Sensitivity analysis, Intervention strategies, Public policy

1. Introduction
Corruption is a pervasive and destructive challenge that undermines governance, public trust, and economic development worldwide. It is particularly severe in sub-Saharan Africa, where weak institutions, fragile regulatory frameworks, and socio-economic vulnerabilities create fertile conditions for corrupt practices (Jain, 2019; Mishra, 2020). Corruption affects both public and private sectors, obstructing equitable resource allocation and hindering economic growth (Ebenezer, 2021; Mishra, 2020). Jain (2019) identifies three primary drivers of corruption: economic incentives, discretionary administrative powers, and weak enforcement of punitive measures. Similarly, Mishra (2020) conceptualizes corruption as an evolving social process that adapts to prevailing socio-economic and institutional conditions. Historically, corruption has persisted across societies and eras, particularly where democratic institutions are weak and public office is exploited for personal gain (Ebenezer, 2021; Bardhan & Mookherjee, 2021). Its consequences include social inequality, mismanagement of resources, weakened public trust, and erosion of justice systems, destabilizing governance and economic development.

Mathematical modeling has increasingly been used to understand and mitigate corruption. Since the 1970s, scholars have developed frameworks to capture the spread, persistence, and social consequences of corrupt behavior (Ebenezer, 2021; Nathan & Jakob, 2021). Epidemiological models, inspired by infectious disease research, have been particularly effective in describing how corruption propagates through communities and institutions (Jain, 2019; Mishra, 2020; Eguda et al., 2019). For example, Eguda et al. (2019) suggested that raising minimum wages could reduce corruption in public procurement, while Lemecha (2020) highlighted thsse effectiveness of awareness campaigns and counseling for incarcerated offenders. However, conventional integer-order models often fall short because they do not effectively capture memory-dependent processes or the long-term dynamics inherent in police and judiciary systems. Fractional calculus provides a more advanced approach, capable of modeling hereditary effects, long-range dependencies, and complex interactions inherent in social systems (Atangana, 2021; Caputo, 1971; Atangana & Baleanu, 2021). Operators such as Caputo-Fabrizio and Atangana-Baleanu enhance modeling precision, making fractional derivatives especially suitable for institutional corruption studies.

Corruption in law enforcement and judicial institutions presents unique challenges. Police corruption, including bribery, coercion, and collusion,is often driven by organizational culture, inadequate accountability, and insufficient training (Transparency International, 2019; Prenzler et al., 2020). Judicial corruption, such as favoritism, political interference, and bribery, undermines investment, economic growth, and public confidence in the rule of law (Bardhan & Mookherjee, 2021; Glaeser & Saks, 2020). Existing research often overlooks the complex interactions among police, judiciary personnel, politicians, citizens, and criminal networks, as well as the long-term dependencies inherent in corrupt systems.

To address these gaps, this study develops a framework integrating fractional-order differential equations (FODEs) with optimal control theory to model corruption dynamics in police and judiciary systems. The framework aims to (i) capture interactions among institutional actors, (ii) incorporate memory effects and long-range dependencies, and (iii) design optimal, resource-efficient interventions. By combining the predictive capabilities of fractional calculus with the strategic insights of optimal control, the model provides evidence-based guidance for policymakers, enhancing transparency, accountability, and societal stability.

Previous research supports the utility of fractional-order modeling and optimal control. Li et al. (2018) and Wang et al. (2020) applied FODEs to government and urban governance systems, demonstrating their ability to capture complex social dynamics. Zhao et al. (2019) illustrated the potential of optimal control for designing interventions in government agencies, while Nwajeri and Omame (2023) showed that lower fractional orders increase population susceptibility to corruption, emphasizing the importance of awareness campaigns. Mokaya et al. (2021) modeled adolescent moral corruption, and Ngondiep (2024) and Aloke (2023) examined poverty-driven corruption using SEIR-type models, applying Pontryagin's maximum principle to design effective interventions.

Additional studies applied fractional derivatives to financial crime and corruption. Zarin and Raezah (2023), Awadalla and Arab (2023), and Abdul Wasa'a and Abd Eljawad (2024) demonstrated predictive and stability analysis capabilities. Bonyah (2020) showed that coordinated, multi-pronged interventions using the Atangana–Baleanu–Caputo derivative are most effective. Arora et al. (2023) and As and Yan (2021) modeled law enforcement and judiciary dynamics to identify key intervention parameters. Research by Doko et al. (2023), Partohaghighi et al. (2022), and Kotola and Teklu (2022) explored social media, racism, and criminal networks' influence, highlighting the sensitivity of corruption dynamics to initial conditions, societal interactions, and intervention strategies.

Despite these advances, gaps remain. Many models focus narrowly on single interventions and fail to capture nonlinear, memory-dependent interactions among stakeholders in police and judiciary systems. Integration of fractional-order modeling with optimal control is limited, particularly for multi-layered dynamics involving police, judiciary, prisoners, citizens, and political actors. Addressing these gaps is essential for designing realistic, effective, and resource-efficient anti-corruption strategies (Transparency International, 2021; World Bank, 2020). This study provides a framework to guide evidence-based policy, strengthen institutional accountability, and support sustainable social development.

2. Model formulation

The model illustrates how corruption spreads through society and institutions. Initially, honest individuals may become influenced and turn corrupt, while some are arrested, punished, or rehabilitated. Corrupt actors affect others, yet decline through legal measures or natural attrition. Honest police and judges can be compromised, reformed, or leave, highlighting the ongoing interplay between societal pressures, institutional integrity, and opportunities for recovery. Thus,

$$
\left.
\begin{aligned}
&{}_{0}^{C}D_t^{\alpha}S(t)=\Lambda-\beta_1 SC(t)-(\alpha_1+\alpha_2+\mu)S(t)\\
&{}_{0}^{C}D_t^{\alpha}E(t)=\beta_1 SC(t)-(\beta_2+\beta_3+\mu)E(t)\\
&{}_{0}^{C}D_t^{\alpha}C(t)=\beta_2 E(t)-(\beta_3+\mu)C(t)\\
&{}_{0}^{C}D_t^{\alpha}I(t)=\beta_3 C-(\beta_4+\mu)V\\
&{}_{0}^{C}D_t^{\alpha}R(t)=\beta_4 V-\mu R\\
&{}_{0}^{C}D_t^{\alpha}J_h(t)=\alpha_1 S-\beta_7 VJ_h-\beta_9 CJ_h-\mu J_h\\
&{}_{0}^{C}D_t^{\alpha}Jc(t)=\beta_7 VJ_h+\beta_9 CJ_h-\mu J_C\\
&{}_{0}^{C}D_t^{\alpha}P_h(t)=\alpha_2 S(t)-\beta_{10}P_h C(t)-\beta_{11}VP_h-\mu P_h(t)\\
&{}_{0}^{C}D_t^{\alpha}P_c(t)=\beta_{10}P_h C(t)+\beta_{11}VP_h-\mu P_c(t)
\end{aligned}
\right\}
\tag{1}
$$

See table 1 and 2 below for state variables and parameter Description and values

Analysis of the Model

The model ensures corruption control measures in police and judiciary are effective, keeping all subpopulation solutions positive and biologically feasible for $t \geq 0$

The Positive Invariant Region

The total population sizes are $N = S(t)+E(t)+C(t)+R(t)+P_h(t)+P_c(t)+J_h(t)+J_c(t)+I(t)$ all variables stay nonnegative over time because zero-valued variables have nonnegative derivatives.

Invariant Region:

Let $N = S(t)+E(t)+C(t)+R(t)+P_h(t)+P_c(t)+J_h(t)+J_c(t)+I(t)$ be the total human population. The total derivatives satisfied

$$D_0^C N = \Lambda - \mu N(t) \tag{2}$$

Solving this inequality yields:

$$0 \le N(t) \le \frac{\Lambda_h}{\mu}, \tag{3}$$

Meaning the population is bounded and eventually remains below this limit.

Feasible Region:

Therefore, all biologically meaningful solutions lie within the compact, positively invariant region:

$$\Omega = \left\{ \left(S,E,C,I,R.J_h,J_c,P_h,P_c\right) \ge 0 : N(t) \le \frac{\Lambda}{\mu} \right\}.$$

is positively invariant.

Equilibrium States

$$E_0^c = (S,E,C,I,J_h,J_C,P_h,P_C) = \left(\begin{array}{l} \dfrac{\Lambda}{(\alpha_1+\alpha_2+\mu)},0,0,0,\dfrac{\alpha_1\Lambda}{(\alpha_1+\alpha_2+\mu)(\beta_1+\mu)},0, \\ \dfrac{\Lambda\alpha_2}{\mu^2+\mu\alpha_1+\mu\alpha_2+\beta_2(\mu+\alpha_1+\alpha_2)},0 \end{array} \right)$$

(4)

Endemic Equilibrium (EE)

$\left(S,E,C,I,R.J_h,J_c,P_h,P_c\right) \neq 0$ Establish once infection persists in the population

Basic Reproduction Number R_o

$$\Re_0 = \frac{\beta_1\Lambda\beta_2}{(\alpha_1+\alpha_2+\mu)(\beta+\mu)(\beta_3+\mu)} \tag{5}$$

Table 1 State variables of the model

Variables	Description	Values	Ref
$S(t)$	Susceptible individuals	0.9527	Mataru & Malonza (2023)
$E(t)$	Exposed individuals	3.823	Mataru & Malonza (2023)
$C(t)$	Corrupt individuals	0.1344	Mataru & Malonza (2023)
$I(t)$	Convicted corrupt individuals	0.5097	Mataru & Malonza (2023)
$R(t)$	Recovered individuals from corruption	0.025	Mataru & Malonza (2023)page 11
$P_h(t)$	Honest judiciary	0.0162	Mataru & Malonza (2023) page 11
$J_h(t)$	Corrupt judiciary	0.2485	Mataru & Malonza (2023) page 11
$P_c(t)$	Corrupt police	0.001	Mataru & Malonza (2023) page11
$J_c(t)$	Corrupt police	0.001	Mataru & Malonza (2023) page 11

Table 2 Parameters of the model

Par.	Description	Values	Reference
β_{12}	Interaction between honest judiciary	0.096.	Assumed
β_{13}	Interaction between honest police	0.039	Assumed
β_1	Transition from susceptible to exposed	0.5800	Partohaghighi & Akgül, A. (2022)
β_2	Transition from exposed to corrupt individual	0.001	Partohaghighi & Akgül, A. (2022)
β_3	Transition from corrupt to convicted individual	0.1265	Partohaghighi & Akgül, A. (2022)
β_4	Transition from convicted to recovery	0.004	29
β_5	Transition from recovery back to exposed	0.44	Assumed
β_6	Interaction between corrupt and honest judiciary	0.015	Partohaghighi & Akgül, A. (2022)
β_7	Interaction between corrupt and honest police	0.5	Partohaghighi & Akgül, A. (2022)
β_8	Interaction between corrupt and corrupt judiciary	0.015	Assumed
β_9	Interaction between corrupt and corrupt police	0.35	Partohaghighi & Akgül, A. (2022)
β_{10}	Interaction between convicted and corrupt police	0.1265	Partohaghighi & Akgül, A. (2022)
β_{11}	Interaction between convicted and corrupt police	0.54	Partohaghighi & Akgül, A. (2022)
μ	Death rate	0.115	Partohaghighi & Akgül, A. (2022)
α_1	First control measure	1	Zhao et al. (2019)
α_2	Second control measure	0.9	Zhao et al. (2019)
α_3	Third control measure	0.8	Zhao et al. (2019)
α_4	Fourth control measure	0.7	Zhao et al. (2019)

2.1 Model Formulation with Control Strategies

To identify the most effective strategies for controlling corruption, it is essential to consider control measures that vary over time. Drawing on the findings of the sensitivity analysis, the following control strategies will be examined:

1. Sensitization of the Masses: This involves using t he media, as well as supporting training programs and public awareness campaigns, to foster a culture of integrity and reduce corruption at all levels of society.
2. Public Accounts Committee Education: Educating the public about the dynamics of corruption within ssssss, empowering citizens with knowledge, and creating a more transparent environment to combat corruption.
3. Punishment of Corrupt Individuals: When police officers, members of the judiciary, and other key players in the enforcement systems are not held accountable for their actions, corruption tends to proliferate and become deeply embedded within these institutions. As noted by Prenzer (2009), consistent and visible punishment of corrupt individuals is crucial in deterring corrupt activities."

However, to build the optimal control model we consider that the susceptible population gets corrupted after getting contact at a rate $1-u_2\beta_1 CS$.and $1-u_1\beta_1 CS$. $0 \le u_1 \le 1$ is the control on the public sensitization,$1-u_2\beta_1 CS$ $0 \le u_2 \le 1$ is the control on the use of public education of the masses $1-u_3\beta_1 CS$. is the control by punishment. Thus $1-u_2$.is educating the masses about corruption.

By bringing together the preceding formulations and assumptions, the model is expressed as the following system of ordinary differential equations:

$$\left.\begin{aligned}
\frac{dS}{dt} &:= \Lambda-(1-u_1)\beta_1 SC-(\alpha_1+\alpha_2+\mu)S\\
\frac{dE}{dt} &:= (1-u_1)\beta_1 SC-((1-u_1)\beta_2+(1-u_2)\beta_3+\mu)E\\
\frac{dC}{dt} &:= (1-u_1)\beta_2 E-((1-u_2)\beta_3+\mu)C\\
\frac{dV}{dt} &:= (1-u_3)\beta_3 C-((1-u_2)\beta_4+\mu)V\\
\frac{dR}{dt} &:= (1-u_2)\beta_4 V-\mu R\\
\frac{dJ_h}{dt} &:= S\alpha_2-(1-u_3)\beta_7 VJ_h-((1-u_3)\beta_5 CJ_h-\mu J_h\\
\frac{dJ_c}{dt} &:= (1-u_3)\beta_7 VJ_h+((1-u_3)\beta_8 CJ_h-\mu J_h\\
\frac{dP_h}{dt} &:= S\alpha_2-(1-u_3)\beta_6 P_h C-(\beta_9 u_1+1)VP_h-\mu P_h\\
\frac{dP_c}{dt} &:= CP_h\beta_{10}+VP_h\beta_{11}-\mu P_c
\end{aligned}\right\}$$

(6)

$$Ju_1,u_2,u_3=\int_0^T\left[A_1C+B_1u_1^2+B_2u_2^2+B_3u_3^2\right]\ (7)$$

Corruption management framed as optimal control minimizing burden and effort

Here, **T** denotes the final time, and the coefficients A_1C, B_1, B_2, B_3 are positive weights used to balance the different components of the objective function. Since the objective function **J** is formulated to minimize the number of individuals involved in corruption, an optimal control u_1^*, u_2^*, u_3^* is obtained that achieves this minimum.

$$J(u_1^*,u_2^*,u_3^*)=\min_{u_1^*,u_2^*,u_3^*}\left\{Ju_1,u_2,u_3 / u_1,u_2,u_3\in\Re\right\}\ (8)$$

Where control set

$\Re = u_1^*, u_2^*, u_3^* / u_1 : 0, T \rightarrow 0,1$ Lévesque measurable $i = 1,2,3$

2.2 Pontryagins Maximum Principle

The key conditions that an optimal control must satisfy are derived from Pontryagin's Maximum Principle (Abdullahi et al., 2023). This principle transforms equations (6) and (7) into a pointwise minimization problem by introducing a Hamiltonian function, H with respect to (u_1, u_2, u_3). Using optimal control theory, we define a set of adjoint variables, and the Hamiltonian for this control problem is expressed as follows:

$$\left.\begin{aligned}
&H := (\Lambda-\beta_1SC-(\alpha_1+\alpha_2+\mu)S)E+C(\beta_1SC-(\beta_2+\beta_3+\mu)E)+(\beta_2E-(\beta_3+\mu)C)P_C+(\beta_3C-\\
&(\beta_4+\mu)V)J_c+\beta_1\mu_1^2+\beta_2\mu_2^2+\beta_3\mu_3^2+\lambda_1(\Lambda-(1-u_1)\beta_1SC-(\alpha_1+\alpha_2+\mu)S)+\lambda_2((1-u_1)\beta_1SC-(1-u_1)\beta_2\\
&+(1-u_2)\beta_3+\mu)E+\lambda_3((1-u_1)\beta_2E-((1-u_2)\beta_3+\mu)C)+\lambda_4((1-u_3)\beta_3C-((1-u_2)\beta_4+\mu)V)+\lambda_5((1-u_2)\\
&\beta_4V-\mu R)+\lambda_6(S\alpha_1-(1-u_3)\beta_7VJ_h-(1-u_3)\beta_5CJ_h-\mu J_h)+\lambda_7((1-u_3)\beta_7VJ_h+(1-u_3)\beta_8CJ_h-\mu J_h)\\
&+\lambda_8(S\alpha_2-(1-u_3)\beta_6J_hC-(-\beta_9u_1+1)VP_h-\mu P_h)+\lambda_9(CP_h\beta_{10}+VP_h\beta_{11}-\mu P_c)
\end{aligned}\right\}(9)$$

The control variables u_1, u_2 and u_3 are chosen such that they meet the optimality conditions of the problem.

Stationary values

$$\begin{cases} u_1 = \max\left\{0, \min\left(1, -\dfrac{c\beta_3\lambda_3 + E\beta_3\lambda_2 + I\beta_4\lambda_4 - I\beta_4\lambda_5}{2B_2}\right)\right\} \\ u_2 = \max\left\{0, \min\left(1, -\dfrac{cS\beta_1\lambda_1 + CS\beta_1\lambda_2 - \lambda_8 I\beta_9 P_h - E\beta_2\lambda_2 + E\beta_2\lambda_3}{2B_2}\right)\right\} \\ u_3 = \max\left\{0, \min\left(1, -\dfrac{CJ_h\beta_5\lambda_6 - CJ_h\beta_8\lambda_7 + CP_h\beta_6\lambda_8 + IJ_h\beta_7\lambda_6 - IJ_h\beta_7\lambda_7 - CB_3\lambda_4}{2B}\right)\right\} \end{cases}$$

(10)

Proof: Corollary 1 (Fleming & Rishel, 1975, cited in Abdullahi & Amiru, 2023) ensures optimal control existence; adjoints derived from Hamiltonian differentiation.
The ad joint system computed includes

$$\left.\begin{aligned} &\frac{d\lambda_1}{dt} := -(-\beta_1 C - \mu - \alpha_1 - \alpha_2)E - C^2\beta_1 - \lambda_1(-(1-\mu_1)\beta_1 C - \alpha_1 - \alpha_2 - \mu) - \lambda_2(1-\mu_1)\beta_1 C - \alpha_1\lambda_6 - \alpha_2\lambda_8 \\ &\frac{d\lambda_2}{dt} := -\Lambda + \beta_1 SC + (\alpha_1 + \alpha_2 + \mu)S - C(-\mu_1 - \beta_2 - \beta_3) - P_c\beta_2 - \lambda_2(-(1-\mu_1)\beta_2 - (1-\mu_2)\beta_3 - \mu) - \lambda_3(1-\mu_1)\beta_2 \\ &\frac{d\lambda_3}{dt} := \beta_1 SE - \mu - 2\beta_1 SC + (\beta_2 + \beta_3 + \mu)E - (-\mu - \beta_3)P_c - \beta_3 J_c + \lambda_1(1-\mu_1)\beta_1 S - \lambda_2(1-\mu_1)\beta_1 S - \lambda_3(-(1-\mu_2) \\ &\beta_3 - \mu) - \lambda_4(1-\mu_3)\beta_3 + \lambda_6(1-\mu_3)\beta_5 J_h - \lambda_7(1-\mu_3)\beta_8 J_h + \lambda_8(1-\mu_3)\beta_6 J_h - \lambda_9\beta_{10} J_h \\ &\frac{d\lambda_4}{dt} := -(-\mu - \beta_4)J_c - \lambda_4(-(1-\mu_2)\beta_4 - \mu) - \lambda_5(1-\mu_2)\beta_4 + \lambda_6(1-\mu_3)\beta_7 J_h - \lambda_7(1-\mu_3)\beta_7 J_h \\ &+\lambda_8(-\beta_9\mu_1 + 1)P_h - \lambda_9\beta_{11}P_h \\ &\frac{d\lambda_5}{dt} := \lambda_5\mu \\ &\frac{d\lambda_6}{dt} := -\lambda_6(-(1-\mu_3)\beta_7 V - (1-\mu_3)\beta_5 C - \mu) - \lambda_7(1-\mu_3)\beta_7 V + (1-\mu_3)\beta_8 C - \mu) \\ &\frac{d\lambda_7}{dt} := -\beta_2 C + (\beta_4 + \mu)V \\ &\frac{d\lambda_8}{dt} := -\beta_2 E + (\beta_3 + \mu)C + \lambda_9\mu \end{aligned}\right\} \quad (11)$$

Since the optimal control is specified only by initial conditions, it is necessary to determine the transversality conditions $\lambda_i = 0$ with $i = 1, \ldots\ldots 10$, which correspond to the terminal conditions in the adjoint equations. In the interior of the control set, where $0 \le u \le i,\ for\, i = 1, 2, 3$

$$\left.\begin{aligned} u_1^* &= \max\left\{0, \min(1, \frac{-C\beta_3\lambda_3 + E\beta_3\lambda_2 + V\beta_4\lambda_4 - V\beta_4\lambda_5}{2B_2})\right. \\ u_2^* &= \max\left\{0, \min(1, \frac{-C\beta_1\lambda_1 + CS\beta_1\lambda_2 - \lambda_8\beta_9 VP_h - E\beta_2\lambda_2 + E\beta_2\lambda_3}{2B_1})\right. \\ u_3^* &= \max\left\{0, \min(1, \frac{-CJ_h\beta_5\lambda_6 - CJ_h\beta_8\lambda_7 + \lambda_8\beta_6 P_h C + VJ_h\beta_7\lambda_6 - VJ_h\beta_7\lambda_7 - C\beta_3\lambda_4}{2B_3})\right. \end{aligned}\right\} \quad (12)$$

By applying standard control theory and taking into account the bounds on the control variables, it can be concluded that (12)

$$
\left.\begin{aligned}
0 &= \frac{\partial H}{\partial u_1} = 2\beta_1 u_1 - (\lambda_8 I_a + \lambda_2 I_h) \\
0 &= \frac{\partial H}{\partial u_2} = 2\beta_2 u_2 - C_8(-S_v\beta_8\lambda_7 + S_v\beta_8\lambda_8 + \lambda_{10}) \\
0 &= \frac{\partial H}{\partial u_3} = 2\beta_2 u_3 - C_8(E_a\beta_4\lambda_5 - S_a\beta_4\lambda_4 - S_h\beta_1\lambda_1 - S_h\beta_1\lambda_2)
\end{aligned}\right\}
$$

$$
\left.\begin{aligned}
u_1^* &= \begin{cases} 0 & if\ \ \xi_1^* \le 0 \\ \xi_1^* & if\ \ 0 < \xi_1^* < 1 \\ 1 & if\ \ \xi_1^* \ge 1 \end{cases} \\
u_2^* &= \begin{cases} 0 & if\ \ \xi_2^* \le 0 \\ \xi_2^* & if\ \ 0 < \xi_2^* < 1 \\ 1 & if\ \ \xi_2^* \ge 1 \end{cases} \\
u_3^* &= \begin{cases} 0 & if\ \ \xi_3^* \le 0 \\ \xi_3^* & if\ \ 0 < \xi_3^* < 1 \\ 1 & if\ \ \xi_3^* \ge 1 \end{cases}
\end{aligned}\right\}
$$

(

(14)

Where, $u_1^* = \min\left\{1, \xi_1^*\right\},\ u_2^* = \min\left\{1, \xi_2^*\right\}, u_3^* = \min\left\{1, \xi_3^*\right\}.$
(15)

Next, we discuss the numerical solutions of the optimality system, examine the corresponding optimal control pair, review the chosen parameter values, and interpret the results across different scenarios.

2.3 Numerical analysis

This section numerically evaluates optimal control effects on corruption using state-adjoint equations and iterative Runge-Kutta methods (Abdullahi & Amiru, 2023; Lenhart & Workman, 2007).

Strategy A: Combination of sensitization and education of the public about corruption

- Strategy B: sensitization of the masses and punishment of the corrupt individuals

- Strategy C: educating the masses and punishment of corrupt individuals

- Strategy D: sensitization, education and punishment of those guilty of corruption

These strategies are analyzed by applying two at a time or all three together to assess their impact on corruption spread.

For the numerical simulation the following weight factors are use $A_i, i = 1,2,3$ *and* $B_i, i = 1,2,3 = 0$

and use the parameter values from Table 1. Other parameter values are in Table 2 to illustrate the effect of different optimal.

3. Optimal Use of the Combination of Sensitization and Education of the Masses (Strategy A)

In this strategy, both the sensitization of the public and the education of the masses are utilized to optimize the objective function, while the punishment control is set to zero. Figure 1 to 6 illustrates the results, showing a significant difference between the system's behaviors with the optimal control strategy compared to when no control is applied.

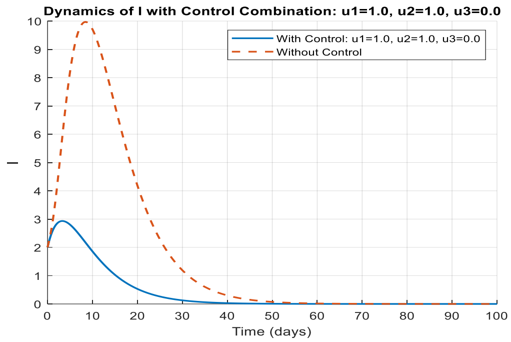


Figure 3: The Plot Shows the Effect Of The Control Strategy A On Convicted Humans

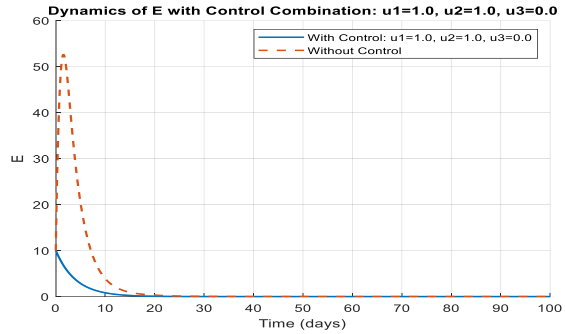


Figure 4: The Plot Shows the Effect Of The Control Strategy A On Exposed Humans

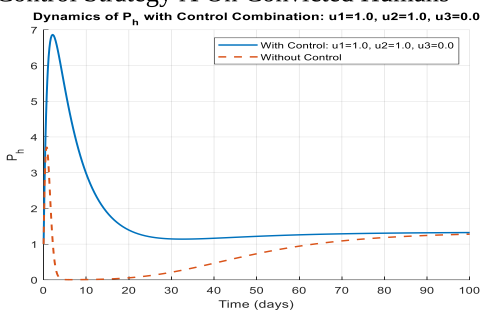


Figure 5: The Plot Shows the Effect of The Control tegy A On Honest Police

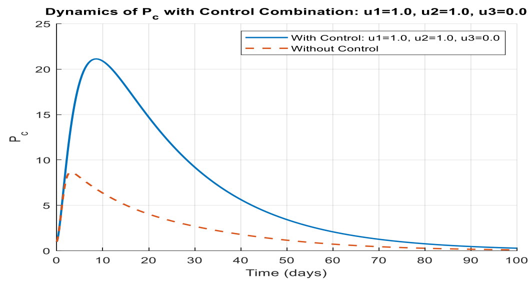


Figure 6: The Plot Shows the Effect of the Control Strategy A On Corrupt Police

Optimal Use of Sensitization and P Stra**unishment (Strategy B)**

In this strategy, both sensitization and punishment are employed to optimize the objective function, while the education of the masses is set to zero. Figure 7 to 15 presents the results, showing a significant difference in the system's dynamics with the optimal control strategy compared to when no control is applied.

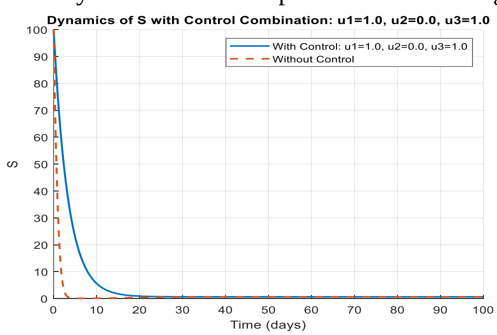
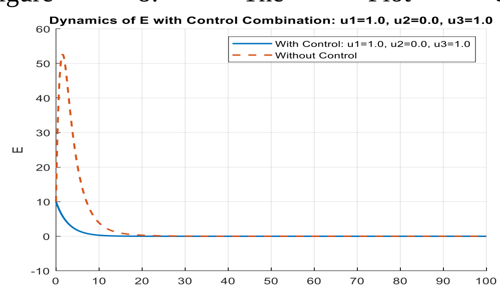


Figure 7: The Plot Shows The Effect Of The Control Strategy B On Susceptible Humans.

Figure 8: The Plot Shows



The Effect Of The Control Strategy B On Exposed Humans

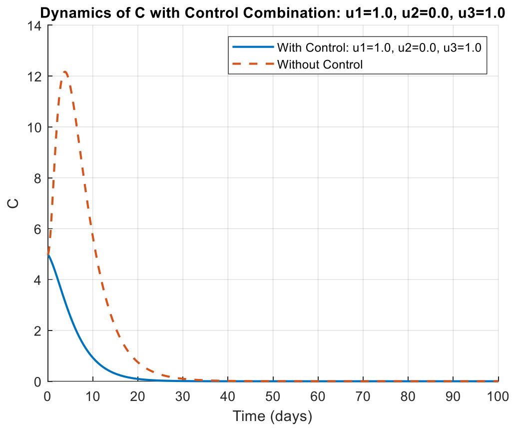


Figure 9: The Plot Shows The Effect Of The Control Strategy B On Corrupt Humans

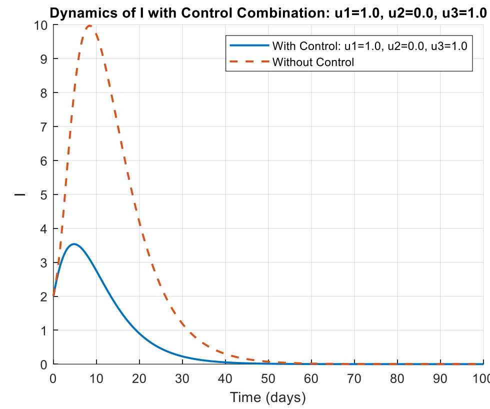


Figure 10: The Plot Shows The Effect Of The Control Strategy B On Convicted Humans

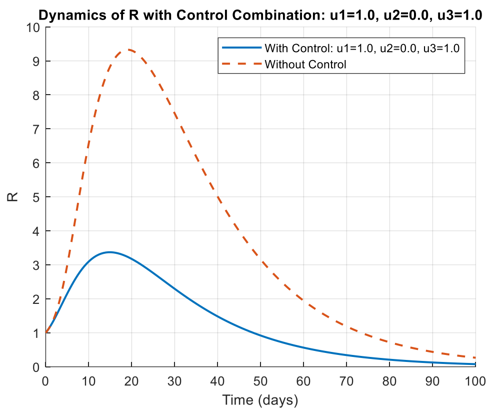


Figure 11: The Plot Shows The Effect Of The Control Strategy B On Recovered Humans

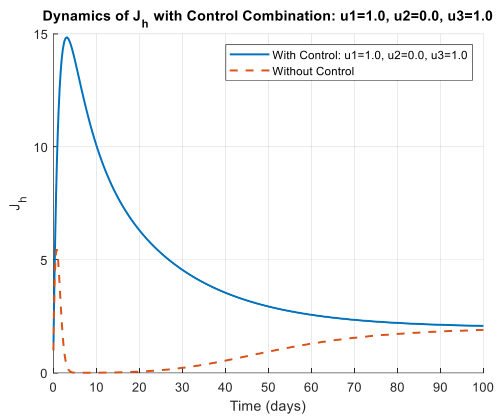


Figure 11: The Plot Shows The Effect Of The Control Strategy B On Honest Judiciary

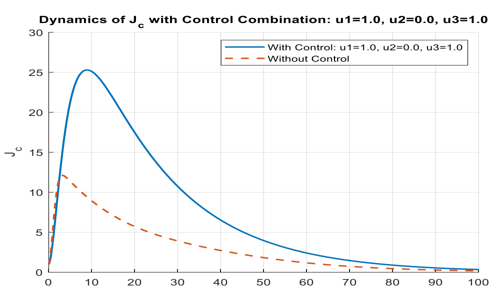


Figure 13: The Plot Shows the Effect of The Control Strategy B On Susceptible Humans Population

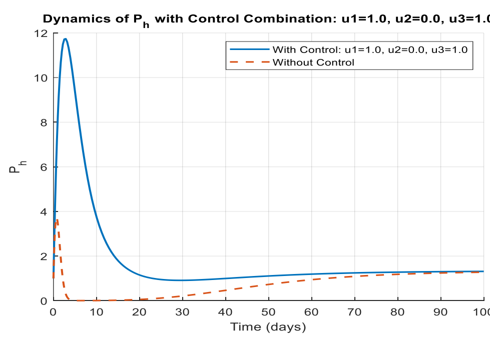
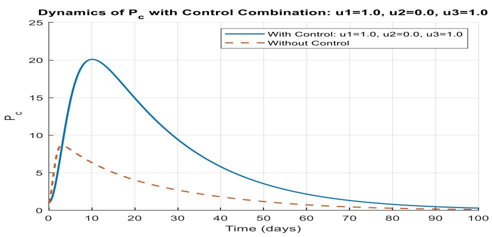


Figure 14: The Plot Shows the Effect Of The Control Strategy B On Susceptible Humans Population



Optimal Use of the Combination of Education Control and Punishment (Strategy C)

In this strategy, both education control and punishment are employed to optimize the objective function, while the sensitization control is set to zero. Figure 16 to 24 presents the results, showing a significant difference in the system's dynamics with the optimal control strategy compared to when no control is applied.

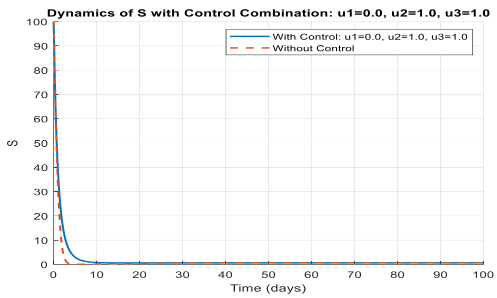


Figure 16: The Plot Shows The Effect Of The Control Strategy C On Susceptible Humans

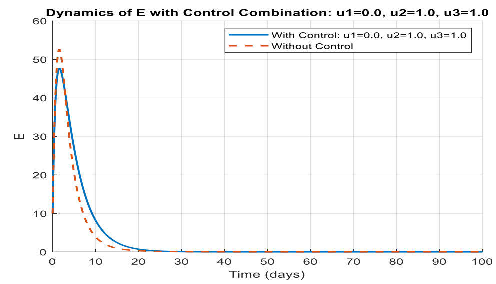


Figure 17: The Plot Shows The Effect Of The Control Strategy C On Exposed Humans

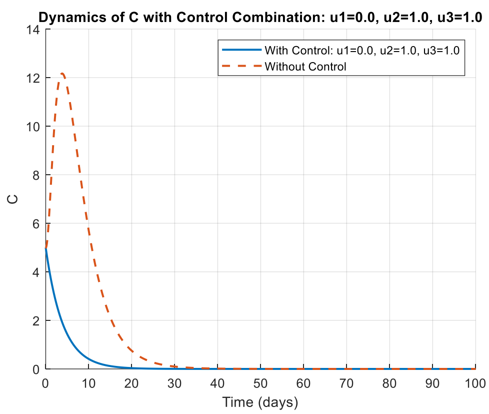


Figure 18: The Plot Shows the Effect of The Control Strategy C On Corrupt Humans

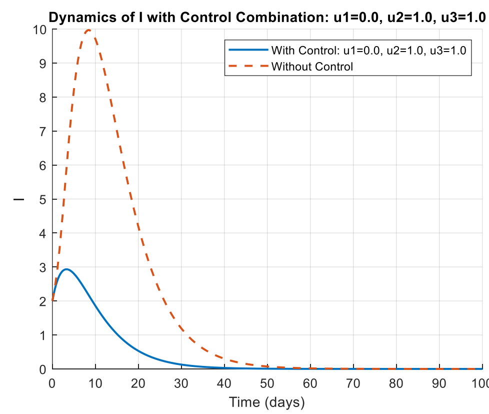


Figure 19: The Plot Shows the Effect of The Control Strategy C On Convicted Humans

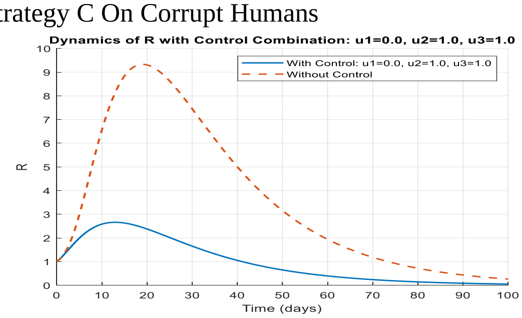


Figure 20: The Plot Shows the Effect of the Control Strategy C On Recovered Humans

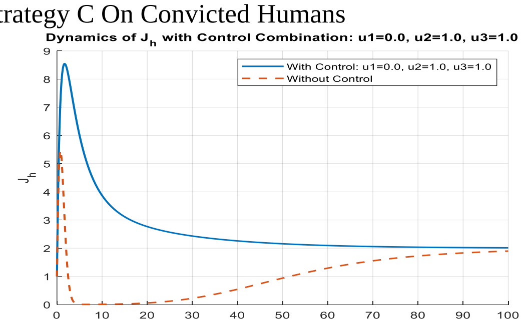


Figure 21: The Plot Shows the Effect of the Control Strategy C On Honest Judiciary

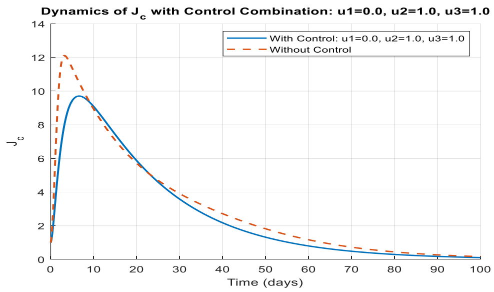


Figure 22: The Plot Shows the Effect of the Control Strategy C On Corrupt Judiciary

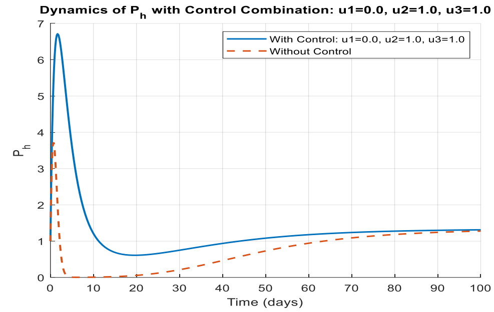


Figure 23: The Plot Shows the Effect of The Control Strategy C On Honest Police

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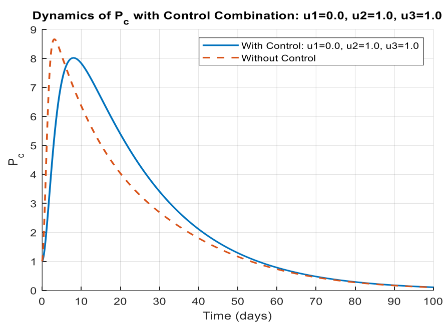

Figure 24: The Plot Shows the Effect of The Control Strategy C On Corrupt Police

Sensitization Control, Education Control, and Punishment Strategy (D)

In this strategy, all the control measures, sensitization control, education control, and punishment are used to optimize the objective function. In Figure 25 to 33, the results demonstrate a significant difference in the system's behavior with the optimal control strategy compared to when no control is applied.

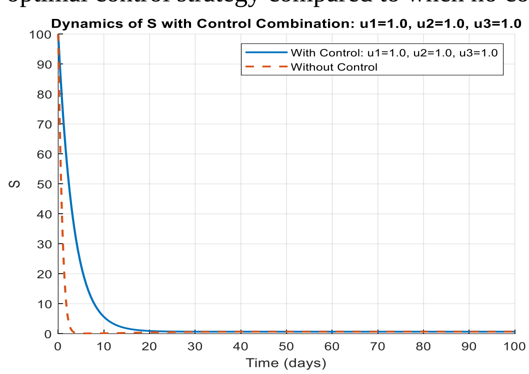

Figure 25: The plot shows the effect of the control strategy D on Susceptible Humans

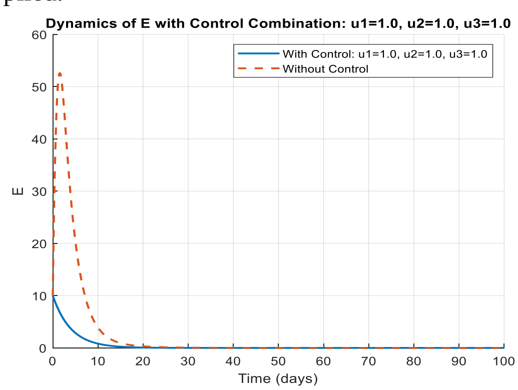

Figure 26: The plot shows the effect of the control strategy C on Exposed human

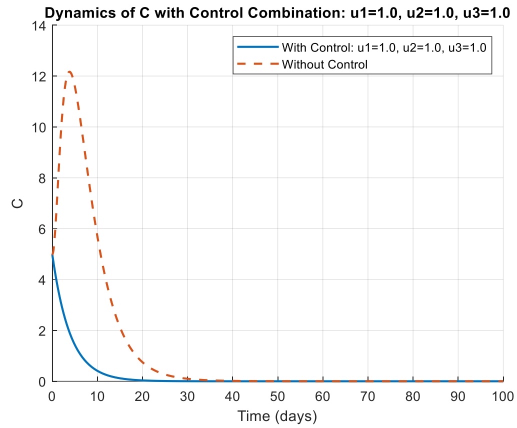

Figure 27: The plot shows the effect of the control strategy C on Corrupt humans

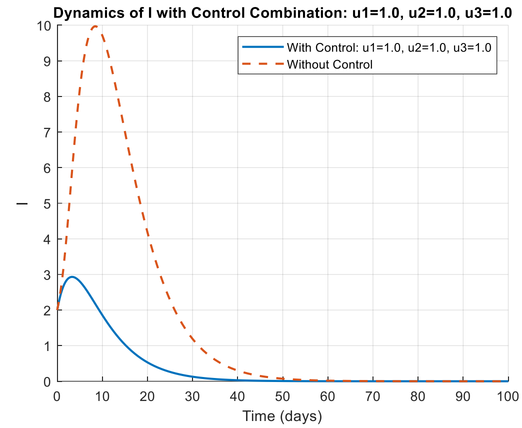

Figure 28: The plot shows the effect of the control strategy C on Convicted humans

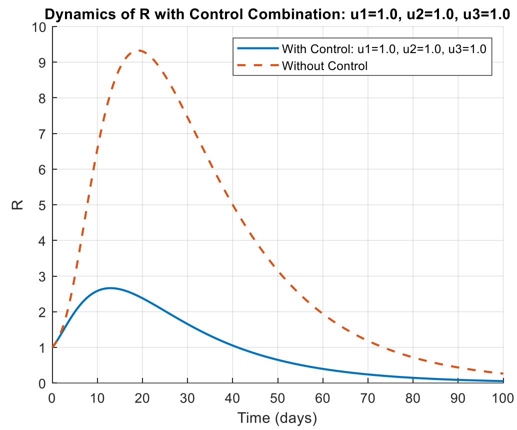

Figure 29: The plot shows the effect of the control strategy C on Recovered humans

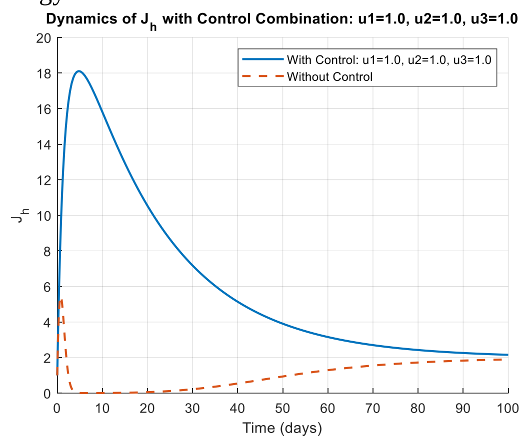

Figure 30: The plot shows the effect of the control strategy C on Honest Judiciary

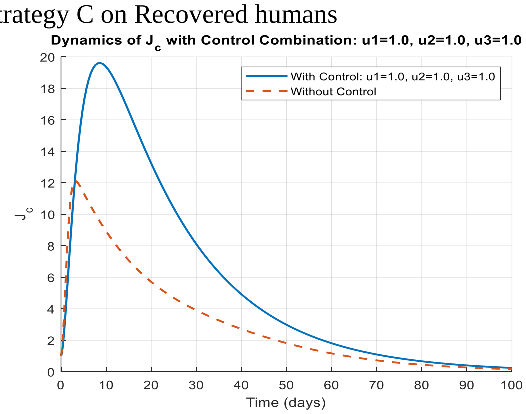

Figure 31: The plot shows the effect of the control strategy C on Corrupt judiciary

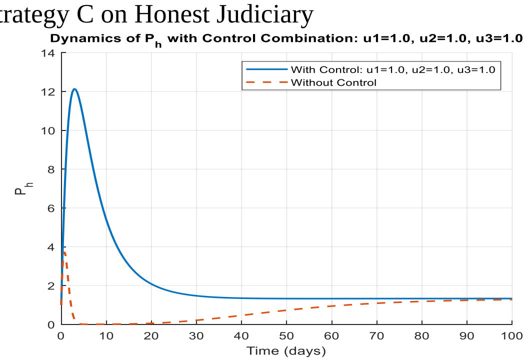

Figure 32: The plot shows the effect of the control strategy C on Honest police

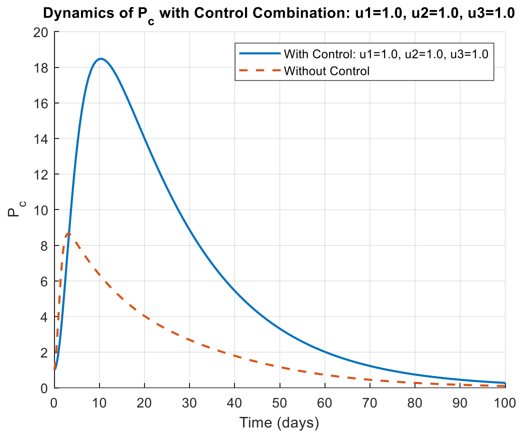

Figure 33: The plot shows the effect of the control strategy C on Corrupt Police

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4. Conclusion

The results revealed that all control combinations effectively reduced corruption within the police and judiciary sectors. However, Strategy D, which integrates sensitization, education, and punishment, produced the most sustained and comprehensive reduction across all compartments, significantly increasing the number of honest officials. Under limited resources**,** Strategy B, sensitization and punishment, proved to be the most practical and efficient option, offering strong corruption control with lower implementation costs. Overall, the study demonstrates that fractional-order, multi-pronged strategies provide policymakers with effective, adaptive, and evidence-based tools for enhancing institutional integrity and accountability.

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